Advanced Algorithms

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- You are running a Hot Tub production company.
- You can produce two types of hot tubs: Aqua-Spas and Hydro-Luxes.
- They require resources (pumps, labor, and tubing), and yield a certain profit

	Aqua-Spa	*Hydro-Lux*
Pumps	*1*	*1*
Labor	*9 hours*	*6 hours*
Tubing	*12 feet*	*16 feet*
Price	*$350*	*$300*

- You have 200 pumps, 1566 hours of labor, and 2880 feet of tubing.
- How many of each hot tub to produce if we want to maximize sales?

Linear Programming is in P

$$
\begin{aligned}
\text{max/min} \quad & r_1x_1 + r_2x_2 + r_3x_3 + \cdots + r_nx_n \\
\text{subject to} \quad & a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n \leq u \\
& \vdots \\
& b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_nx_n \geq v \\
& \vdots \\
& c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n = w \\
& \vdots
\end{aligned}
$$

Integer Linear Programming . . .

$$\begin{aligned}
\text{max/min} \quad & r_1x_1 + r_2x_2 + r_3x_3 + \cdots + r_nx_n \\
\text{subject to} \quad & a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n \leq u \\
& \vdots \\
& b_1x_1 + b_2x_2 + b_3x_3 + \cdots + b_nx_n \geq v \\
& \vdots \\
& c_1x_1 + c_2x_2 + c_3x_3 + \cdots + c_nx_n = w \\
& \vdots
\end{aligned}$$

x_i is an integer value for all $i \in [n]$

Is NP-hard

Why?

Integer Programming formulation:

x = number of Aqua-Spas to produce
y = number of Hydro-Luxes to produce

Maximize: $350x + 300y$

Subject to:

$x + y \leq 200$	(pumps)
$9x + 6y \leq 1566$	(labor)
$12x + 16y \leq 2880$	(tubing)
$x, y \geq 0$	(non-negativity)
x,y integer	

Linear Programming relaxation:

x = number of Aqua-Spas to produce
y = number of Hydro-Luxes to produce

Maximize: $350x + 300y$

Subject to:

$x + y \leq 200$	(pumps)
$9x + 6y \leq 1566$	(labor)
$12x + 16y \leq 2880$	(tubing)
$x, y \geq 0$	(non-negativity)
~~x,y integer~~	

How can this be used in an algorithm?

New Scenario

	Aqua-Spa	*Hydro-Lux*
Pumps	*1*	*1*
Labor	*9 hours*	*6 hours*
Tubing	*12 feet*	*16 feet*
Price	*$350*	*$300*

- You have 200 pumps, 1566 hours of labor, and 2880 feet of tubing.
- You have hired exactly 1566 hours, have to pay $8 for over time or idle time.

How to model this?